Temple University

College of Engineering

Department of Electrical and Computer Engineering (ECE)

Student Report Cover Page

Course Number: ECE 3412

Lab 9: Lag Compensator Design

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Grade: / 100

I. Introduction

In this lab we will be experiment with lead compensator design. This is important because lead compensators add poles and zeros to the closed loop transfer function, which in turn alters the shape of the root locus. If we can alter the shape of the root locus, we can ensure that the poles that give us our desired settling time and steady state error lie on the root locus, which leaves us to simply calculate the proportional gain that gives us the desired pole.

II. Procedure

To begin the design of a lead compensator we need to analyze our DC motor system. We build our traditional DC motor apparatus and measure the system response to a 0.5 unit step. Also as before we can estimate our first order transfer function for the data collected using the system identity toolbox built into MatLab. After we have our transfer function we can plot the root locus fairly easily also in Matlab using the 'rlocus' function. The transient response of the will show lots of steady state error. We can find steady state error by:

$$K_P = \lim_{s \to 0} G(s) \rightarrow e_{ss} = \frac{R}{1 + K_P}$$

Now that we have the steady state error we can try to reduce the error by introducing a lead compensator. We pick a new steady state error of 0.25 and find the lead compensator accordingly for when our zeros of our compensator are 0.01, 0.1, and 1. The compensator will change our transfer function so that our new transfer function becomes:

$$CG(s) = C_{Lead(s)} * G(s)$$

For each lead compensator case we plot the root locus and the corresponding transient response. We compare each compensator to determine which produces the best response.

III. Results

Below we see the first order transfer function that describes the open loop uncompensated system.

$$G(s) = \frac{21.54}{s + 23.4}$$

First Order Transfer Function of Open Loop System

For the above transfer function, we have the following root locus diagram.

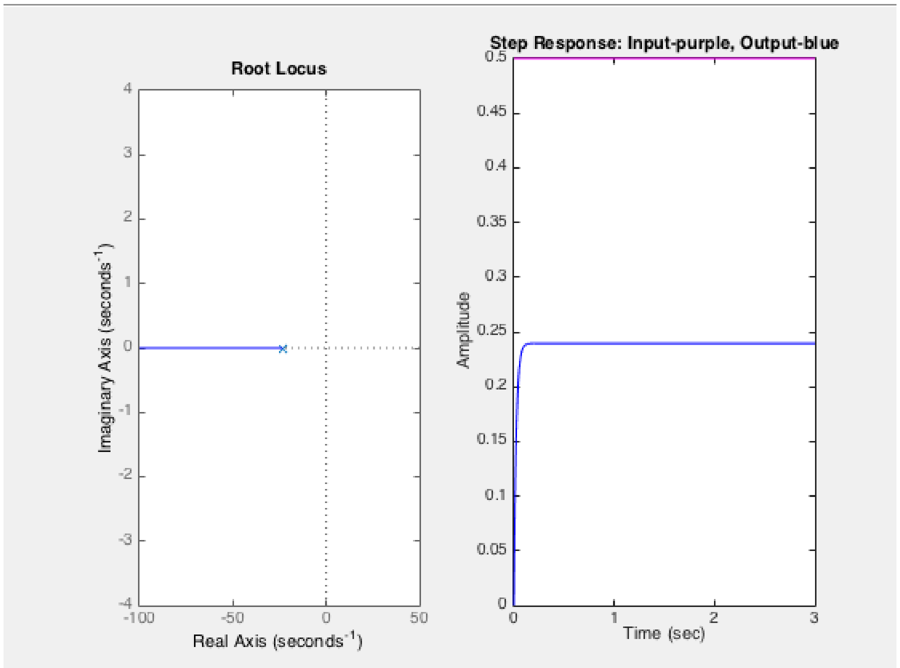


Figure 1: Root Locus and Step Response for First Order Open Loop Transfer Function

It is now important to find k_p for this system. This constant is important because it dictates the steady state error. The steady state error is given in the following equation

$$e_{ss} = \frac{1}{1 + k_p}$$

From this equation, we can see that in order to minimize the steady state error, we have to make k_p large. As it stands, the k_p for the open loop transfer function is $\frac{21.54}{23.4}$. This is found by taking the limit of the open loop transfer function as s approaches 0.

Now using the equation for steady state error, and the value of k_p shown above, we can calculate the steady state error of our system. The calculation yields a steady state error of .26. The simulation of the system is shown below, from which we can verify our steady state error calculation.

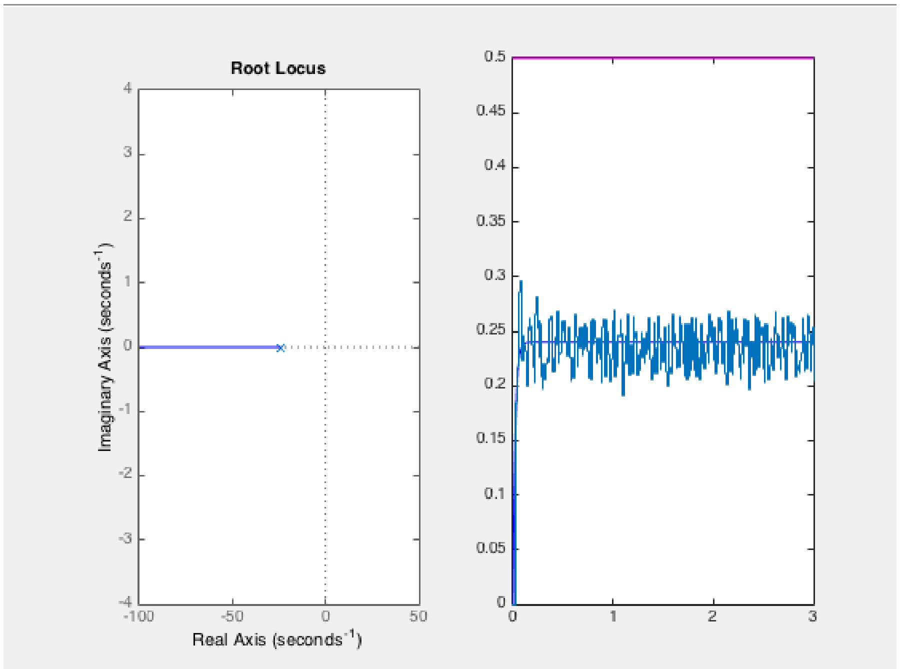


Figure 2: Simulation Results

As we can see from the figure above, the steady state error is 0.26. For the purpose of this lab, the desired steady state error is no more than .05. To reduce the steady state error, we will design a lead compensator. To begin, we first must figure out what k_p is required for the desired steady state error. We can find a new k_p by setting our steady state error equal to a desired value of 0.025 and solving for k_p.

$$\frac{.5}{1+k_p} = .025 \rightarrow k_p = 19$$

We are also interested in the ratio of our new k_p to the old k_p. This ratio is obtained as the following:

$$\frac{k_{pn}}{k_{po}} = \frac{19}{\frac{21.54}{23.4}} = 20.64 = R$$

We will now begin to calculate the pole and zero locations of our compensator. If we choose the pole of the compensator to be .01, we can calculate the required zero location that would yield the desired k_p. The equation is as follows.

$$Z_c = RP_c = 20.64(0.01) = .21$$

We have the following simulation and experimental results.

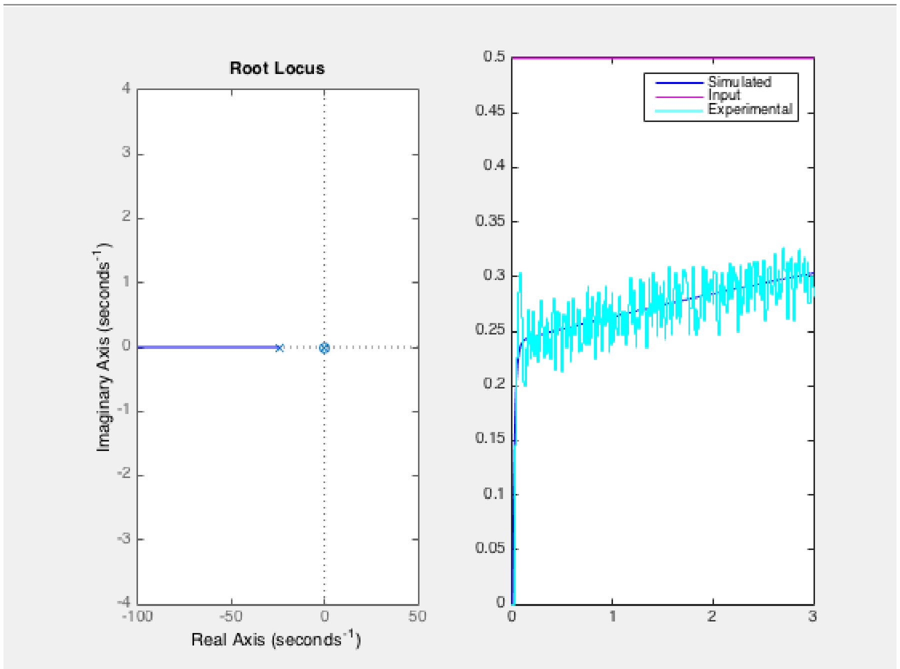


Figure 3: Results for Zc=.01

We see that the simulation results and the experimental results converge. We also see that this does not get to the desired steady error, but the rise time is extremely long. For this reason, we try a larger pole location.

The following is the root locus and response with the new CG(S).

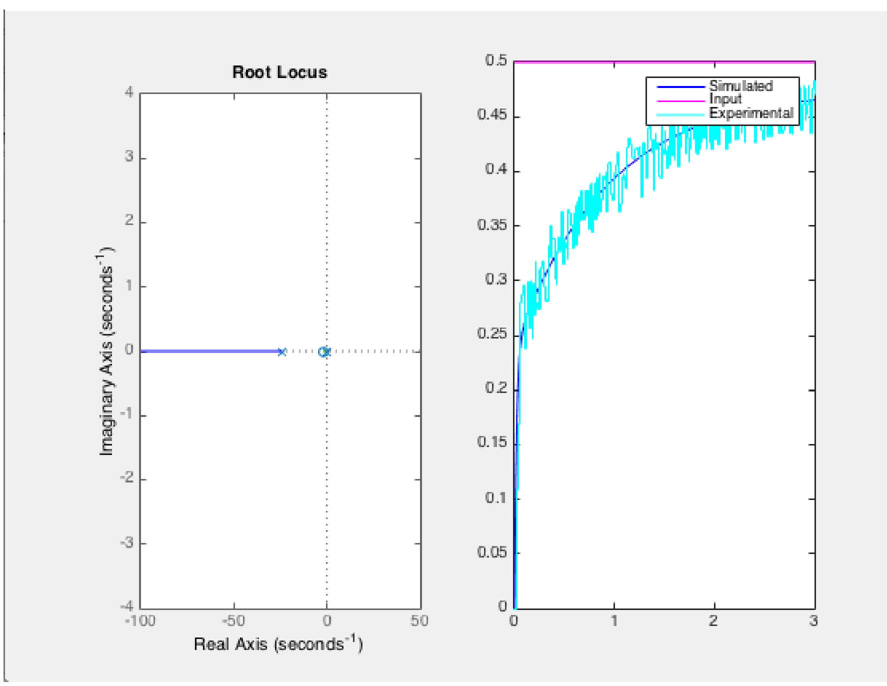


Figure 4: Response for Zc = .1

We can see as we are increasing the pole of the compensator, we are achieving a faster settling time while maintaining the same steady state error.

Following the same process again for a pole location of 1 yields a compensator zero of 20.6, and the output is shown below.

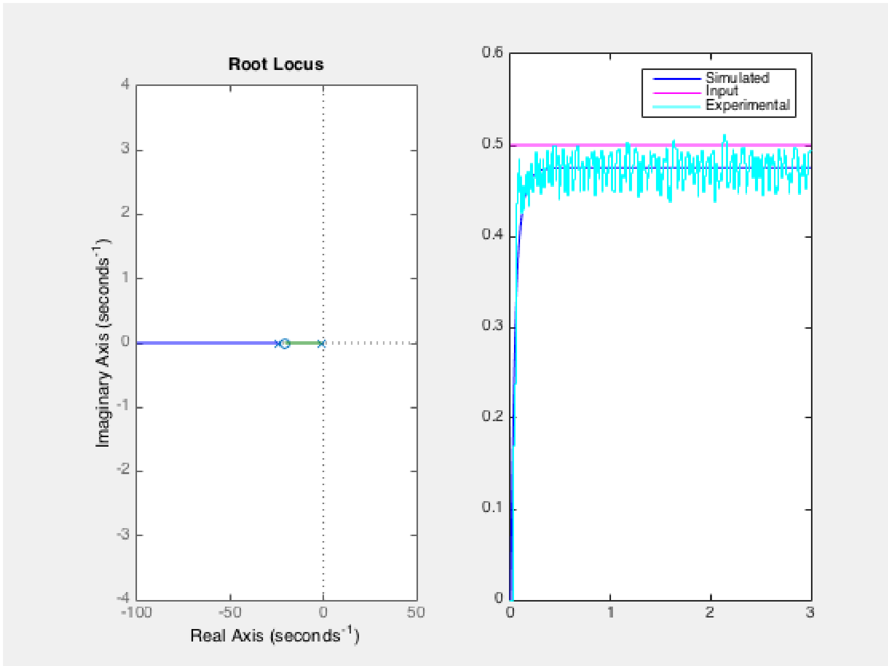


Figure 5: System Response for Zc of 1

We can see that as we increase the pole, the settling time is decreased. This makes sense as the settling time is defined as $4/\zeta\omega_n$ which is the real part of the root.

IV. Discussion

We showed how the different compensators effect the transient response of our original system. This lab was again completely digital thus we question how we would construct a compensator using only analog components? As with the previous lab, we note that compensators can be constructed in similar fashion to PID controllers.

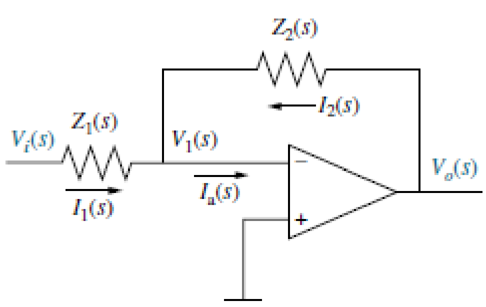
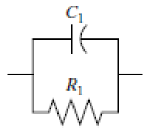
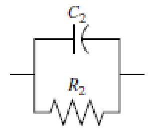
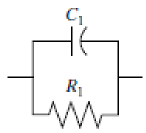
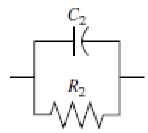


Figure 6: Bias for the Active Circuit Realization

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Lag compensation			$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_2C_2 > R_1C_1$
Lead compensation			$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_1C_1 > R_2C_2$

The circuit calls for using an OP-Amp with Z_1 & Z_2 being a parallel capacitor resistor combination. The values we choose for our capacitor and resistance will determine the lead/lag compensator we construct. We saw in Lab 8 how effective analog circuits can be when constructing PID controllers, and compensators are no different. Analog realization of compensators are cheaper, more reliable, but harder to adjust.

Compensators have thus been shown to adjust our transfer function so that it meets the design criteria we require. With a compensator we can adjust a system by introducing new poles and zeros so that it has a specified percent overshoot or settling time. By using root locus we can analyze how a compensator will alter the transient response of said system. Compensators are useful in real world applications in areas of system stabilization in areas such as satellites, or laser frequency stabilization. The compensator will aid PID controllers in adjusting any system to fit specification.